

**Teaching calculation
strategies that
lead onto written
methods of calculation**

Introduction: The aim is for all children to have one written method for each of the four operations which is reliable and efficient

- Each page refers to a different operation, i.e +, -, x and \div . **Stages refer to the guidance paper on written calculation strategies in the renewed framework.**
- There is progression in calculation from N to Y6.
- In order for progression to run smoothly vocabulary, mental calculation strategies and rapid recall facts should be introduced at the appropriate stage and reinforced.
- Throughout KS1 and KS2 it is assumed that mathematical vocabulary will be introduced and used in context. **Refer to the electronic Renewed framework**
- Numerical examples have been included to illustrate progression from year to year as well as development within the year.
- The different approaches to mental and written calculation should be made explicit.
- It is important to notice consistency between methods of written calculations (+ and – starting with units; when dividing in Y5 and Y6, be consistent with subtraction method needed for children to divide by "chunking", i.e. repeated subtraction), therefore when adding or subtracting in earlier years, begin with the least significant digits.
- Use the Framework for Teaching Mathematics to ensure the appropriate numbers are used for appropriate year groups or levels of attainment.
- Some methods rely heavily on:
 - A firm understanding of place value
 - Interpretation of written signs
 - Familiarity with a range of vocabulary / Efficient mental calculation

Addition

To add successfully, children need to be able to:


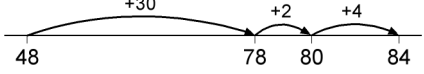
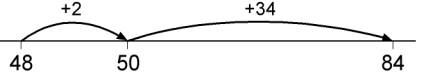
- recall all addition pairs to $9 + 9$ and complements in 10;
- add mentally a series of one-digit numbers, such as $5 + 8 + 4$;
- add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

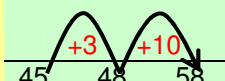
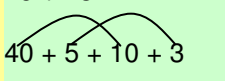
Progression examples:

1. No 'carrying'	$\begin{array}{r} 23 \\ +42 \\ \hline \end{array}$	$\begin{array}{r} 315 \\ +624 \\ \hline \end{array}$
2. Extra digit in answer	$\begin{array}{r} 94 \\ +73 \\ \hline \end{array}$	$\begin{array}{r} 561 \\ +718 \\ \hline \end{array}$
3. Carrying U (units/ones) to T (tens)	$\begin{array}{r} 47 \\ +25 \\ \hline \end{array}$	$\begin{array}{r} 237 \\ +516 \\ \hline \end{array}$
4. Carrying T to H	$\begin{array}{r} 371 \\ +485 \\ \hline \end{array}$	$\begin{array}{r} 293 \\ +541 \\ \hline \end{array}$
5. Carrying U to T and T to H	$\begin{array}{r} 376 \\ +485 \\ \hline \end{array}$	$\begin{array}{r} 295 \\ +547 \\ \hline \end{array}$
6. More than two numbers to be added	$\begin{array}{r} 463 \\ 921 \\ +759 \\ \hline \end{array}$	
7. Different numbers of digits	$\begin{array}{r} 24 \\ 375 \\ + 48 \\ \hline \end{array}$	$\begin{array}{r} 4756 \\ 20375 \\ + 752 \\ \hline \end{array}$
8. Real life problems involving money or measures	$\begin{array}{r} \pounds 4.21 \\ +\pounds 3.87 \\ \hline \end{array}$	$\begin{array}{r} 24.90\text{kg} \\ +7.25\text{kg} \\ \hline \end{array}$
9. Numbers with decimals	$\begin{array}{r} 2.13 \\ +5.62 \\ \hline \end{array}$	

Addition

Pre-stage	Stage 1 The Empty Number Line	Stage 2 Partitioning (by the end of year 3)	Stage 3 Expanded method in columns	Stage 4 Column Method (by the end of Year 4)												
<ul style="list-style-type: none"> Counting on fingers Count on one digit from a two digit number Counting forwards and backwards on a number line Partition into multiples of 10 and 1 and recombine Derive fact families using addition and subtraction <p> $20 - 7 = 13$ $20 - 13 = 7$ $13 + 7 = 20$ $7 + 13 = 20$ </p> <p>Say how a set of objects can be separated into 2 groups 8 is 5 and 3</p>	<p>Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.</p> <p>$8 + 7 = 15$</p>  <p>$48 + 36 = 84$</p>  <p>or:</p>  <p>Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.</p>	<p>Record steps in addition using partitioning:</p> <p>$47 + 76 = 47 + 70 + 6 = 117 + 6 = 123$</p> <p>$47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123$</p> <p>Partitioned numbers are then written under one another:</p> $\begin{array}{r} 47 = 40 + 7 \\ +76 \quad 70 + 6 \\ \hline 110 + 13 = 123 \end{array}$	<p>Practical apparatus to be used when learning carrying and exchanging</p> <p>Write the numbers in columns.</p> <p>Adding the tens first:</p> $\begin{array}{r} 47 \\ + 76 \\ \hline 110 \\ \underline{13} \\ 123 \end{array}$ <p>Adding the ones first:</p> $\begin{array}{r} 47 \\ + 76 \\ \hline 13 \\ \underline{110} \\ 123 \end{array}$ <p>Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to adding the ones digits first consistently.</p>	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">47</td> <td style="text-align: center;">258</td> <td style="text-align: center;">366</td> </tr> <tr> <td style="text-align: center;">$+ 76$</td> <td style="text-align: center;">$+ 87$</td> <td style="text-align: center;">$+ 458$</td> </tr> <tr> <td style="text-align: center;">$\hline 123$</td> <td style="text-align: center;">$\hline 345$</td> <td style="text-align: center;">$\hline 824$</td> </tr> <tr> <td style="text-align: center;">$\hline 11$</td> <td style="text-align: center;">$\hline 11$</td> <td style="text-align: center;">$\hline 11$</td> </tr> </table> <p>Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.</p> <p>Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.</p>	47	258	366	$+ 76$	$+ 87$	$+ 458$	$\hline 123$	$\hline 345$	$\hline 824$	$\hline 11$	$\hline 11$	$\hline 11$
47	258	366														
$+ 76$	$+ 87$	$+ 458$														
$\hline 123$	$\hline 345$	$\hline 824$														
$\hline 11$	$\hline 11$	$\hline 11$														

Addition

N	R	Y1	Y2	Y3 Stage 2	Y4 Stage 4 by the end of year 4	Y5	Y6
<ul style="list-style-type: none"> Through practical activities in meaningful contexts. Counting on fingers in a consistent way <ul style="list-style-type: none"> Partitioning 	say how a set of objects can be separated into 2 groups 8 is 5 and 3 <ul style="list-style-type: none"> counting on using fingers 	Adding multiples of 10. $30 + 20 = 50$ Adding multiples of 10 and units. $10 + 5 = 15$	$70 + 30 = 100$ $42 + 30 = 72$ $45 + 13 = 45 + 3 + 10 = 58$ 	$67 + 24 = 60 + 7 + 20 + 4 = 11 + 80 = 91$	$367 + 85 = 452$ 12 add mentally 140 from top to bottom 300	$7587 + 675 = 8262$ 12 150 1100 7000 8262 (units first)	$7648 + 1486 = 9134$ 14 120 1000 8000 9134 (units first)
		<ul style="list-style-type: none"> combining sets to make a total steps along a number line (counting on) record using + and = signs Add a one digit number to a one of two digit number Stage 1 – number line 	$45 + 13 = 45 + 5 + 10 + 3 = 58$ 	$67 + 24 = 11 + 80 = 91$ 67 + 24 11 (7 + 4) 80 (60 + 20) 91 (units first)	"carrying"	Relies heavily on firm understanding of place value	$3587 + 675 = 4262$ 1111
<ul style="list-style-type: none"> Illustrate number stories with number sentences Model and encourage use of mathematical language <ul style="list-style-type: none"> count on altogether one more, etc. Model interpretation of and sign using appropriate language 	Combine numbers of objects – count all the objects	<ul style="list-style-type: none"> Derive fact families Using add / subtract eg $20 - 7 = 13$ $20 - 13 = 7$ $13 + 7 = 20$ $7 + 13 = 20$ 	Extend to 3 digit numbers	Real life money problems decimals	$£4.21 + £3.87 = £8.08$ 0.08 1.00 7.00 £8.08 3 numbers less than 1000	$72.5 \text{ km} + 54.6 \text{ km} = 127.1 \text{ km}$ 111	$124.90 \text{ kg} + 7.25 \text{ kg} = 132.15 \text{ kg}$ 111
Deliberately count on the wrong number. Ask the children how to put it right.					$£4.21 + £3.87 = £8.08$ 3 numbers less than 1000	Numbers with any amount of digits, e.g. 72 567 26 +	

Addition

367
569
826 +
22
140
1600
Leading to 'carrying'
367
85 +
452
11

Units first. Call each digit by its value, e.g. "60 plus 80 is 150"

* links to Y2 partitioning

Subtraction

To subtract successfully, children need to be able to:

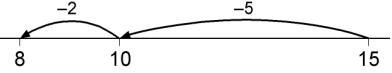
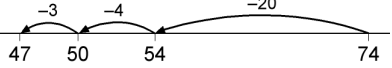
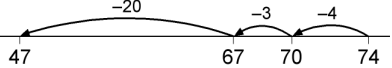
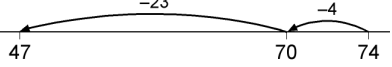
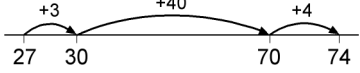
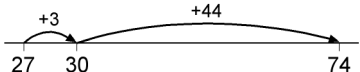
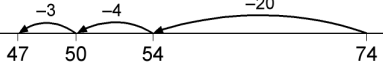
- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as $160 - 70$) using the related subtraction fact, $16 - 7$, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into $70 + 4$ or $60 + 14$).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

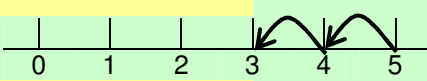
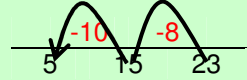
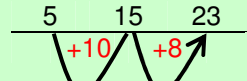
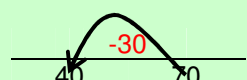
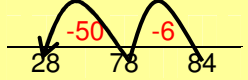
Progression examples:

1. No adjustment	47 <u>-23</u>	864 <u>-621</u>
2. Adjustment T to U	51 <u>-36</u>	432 <u>-217</u>
3. Adjustment H to T	437 <u>-182</u>	618 <u>-217</u>
4. Adjustment H to T and T to U	432 <u>-187</u>	
5. Noughts	470 <u>-142</u>	700 <u>-485</u>
		604 <u>-347</u>

Subtraction

Pre-stage	Stage 1 The Empty Number Line	The Counting Up Method	Stage 2 Partitioning	Stage 3 Expanded method in columns
<ul style="list-style-type: none"> Counting back from a larger number One less, two less Counting on from a smaller number Find the difference by counting up – important to use visual images <hr/> <hr/>	<p>Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.</p> <p>$15 - 7 = 8$</p>  <p>$74 - 27 = 47$ worked by counting back:</p>  <p>The steps may be recorded in a different order:</p>  <p>or combined:</p>  <p>Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.</p>	<p>The Counting Up Method</p>  $\begin{array}{r} 74 \\ - 27 \\ \hline 3 \rightarrow 30 \\ 40 \rightarrow 70 \\ 4 \rightarrow 74 \\ \hline 47 \end{array}$ <p>or:</p>  $\begin{array}{r} 74 \\ - 27 \\ \hline 3 \rightarrow 30 \\ 44 \rightarrow 74 \\ \hline 47 \end{array}$	<p>Subtraction can be recorded using partitioning:</p> $74 - 27 = 74 - 20 - 7 = 54 - 7 = 47$ $74 - 27 = 70 + 4 - 20 - 7 = 60 + 14 - 20 - 7 = 40 + 7$ <p>This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.</p> 	<p>Practical apparatus to be used when learning carrying and exchanging</p> <p>Partitioned numbers are then written under one another:</p> <p>Example: $74 - 27$</p> $\begin{array}{r} 70 + 4 \\ - 20 + 7 \\ \hline 40 + 7 \end{array} \quad \begin{array}{r} 60 \quad 14 \\ - 70 + 4 \\ \hline 40 + 7 \end{array} \quad \begin{array}{r} 6 \quad 14 \\ - 7 \quad 4 \\ \hline 4 \quad 7 \end{array}$ <p>Example: $741 - 367$</p> $\begin{array}{r} 700 + 40 + 1 \\ - 300 + 60 + 7 \\ \hline 300 + 70 + 4 \end{array} \quad \begin{array}{r} 600 \quad 130 \quad 11 \\ - 700 + 40 + 1 \\ \hline 300 + 70 + 4 \end{array} \quad \begin{array}{r} 6 \quad 13 \quad 11 \\ - 7 \quad 4 \quad 1 \\ \hline 3 \quad 7 \quad 4 \end{array}$ <p>The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.</p>

Subtraction

N	R	Y1	Y2	Y3	Y4	Y5	Y6
	counting back	counting back by partitioning			Stage 3 by the end of stage 4 decomposition		
Through practical activities in meaningful contexts "How many are left?" "Take away" "The difference between"		Taking away and count how many are left.	(units first) $23 - 18$ $23 - 8 - 10$	Stage 2 (units first) $84 - 56 = 28$	Stage 3 (units first) $81 - 57 = 24$	(units first) $754 - 286$	(units first)
$5 - 2 = 3$ 	counting back from the larger number one less, etc.	Count back from the larger number.	  $70 - 30 = 40$ 	$84 - 56 = 28$ 	$70 \ 11$ $- \underline{50 \ 7}$ $20 \ 4$	$700 \ 50 \ 4$ $- \underline{200 \ 80 \ 6}$ $600 \ 140 \ 14$ $- \underline{200 \ 80 \ 6}$ $400 \ 60 \ 8$	$6 \ 4 \ 6 \ 7$ $- \underline{2 \ 6 \ 8 \ 4}$ $3 \ 7 \ 8 \ 3$
		Counting on from smaller number. $14 - 12 = 2$ $15 - 8 = 7$ $21 - 10 = 11$		Extend to 3 digit numbers	$754 - 86$ $600 \ 140 \ 14$ $- \underline{ \ 80 \ 6}$	$754 - 286$ $700 \ 50 \ 4$ $- \underline{200 \ 80 \ 6}$ $600 \ 140 \ 14$ $- \underline{200 \ 80 \ 6}$ $400 \ 60 \ 8$ $= 468$	$3 \ 7 \ 8 \ 3$
		multiples of 10 $50 - 20 = 30$			$754 - 86$ $600 \ 140 \ 14$ $- \underline{ \ 80 \ 6}$ $600 \ 60 \ 8$ $= 668$	leading to $6 \ 14 \ 1$ $\cancel{7} \ \cancel{5} \ 4$ $- \underline{2 \ 8 \ 6}$ $4 \ 6 \ 8$	
		Record using – and = signs			extend to decimals using chosen method		
			Using an empty number line		Call each digit by its value. $£6.28$ $- \ £3.15$		
		Find the difference by counting up	Understand inverse is addition $20 - 13 = 7$ $13 - 7 = 20$ Fact families				

Multiplication

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10×10 ;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70×5 , 70×50 , 700×5 or 700×50 using the related fact 7×5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Progression examples:

1. No 'carrying'	32	44
	<u>x3</u>	<u>x2</u>
2. Extra digit	32	51
	<u>x4</u>	<u>x4</u>
3. 'Carrying' but keeping in same decade	83	34
	<u>x4</u>	<u>x7</u>
4. 'Carrying' and going into next decade	78	68
	<u>x7</u>	<u>x8</u>
5. Noughts	202	430
	<u>x4</u>	<u>x6</u>
6. Multiplying by multiples of 10	87	416
	<u>x10</u>	<u>x60</u>
7. 'Long' multiplication	47	832
	<u>x23</u>	<u>x74</u>

Multiplication

Stage 1 Mental multiplication using partitioning	Stage 2 The Grid Method (introduced in year 4)	Stage 3 Expanded short multiplication (by end of year 4)	Stage 4 Short Multiplication	Stage 5 Two-digit by Two-digit products (by end of year 5)																																																								
<p>Informal recording in Year 4 might be:</p> $\begin{array}{r} 43 \\ 40 + 3 \\ \downarrow \quad \downarrow \\ 240 + 18 = 258 \end{array} \times 6$ <p>Also record mental multiplication using partitioning:</p> $14 \times 3 = (10 + 4) \times 3 = (10 \times 3) + (4 \times 3) = 30 + 12 = 42$ $43 \times 6 = (40 + 3) \times 6 = (40 \times 6) + (3 \times 6) = 240 + 18 = 258$ <p>Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:</p> $7 \times 3 = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$	$38 \times 7 = (30 \times 7) + (8 \times 7) = 210 + 56 = 266$ <table border="1" data-bbox="584 416 898 552"> <tr><td>x</td><td>30</td><td>8</td><td></td></tr> <tr><td>7</td><td>210</td><td>56</td><td>266</td></tr> <tr><td></td><td></td><td></td><td></td></tr> </table> $\begin{array}{r} 30 + 8 \\ \times \quad 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$ <p>Derive and recall ALL multiplication facts up to 10 x 10, the corresponding division facts and multiples of numbers to 10 up to the tenth multiple.</p>	x	30	8		7	210	56	266					$\begin{array}{r} 30 + 8 \qquad 38 \\ \times \quad 7 \qquad \times \quad 7 \\ \hline 210 \qquad 30 \times 7 = 210 \qquad 210 \\ 56 \qquad 8 \times 7 = 56 \qquad 56 \\ \hline 266 \qquad \qquad \qquad 266 \end{array}$ <p>Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38×7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3×7 should be stressed.</p>	$\begin{array}{r} 38 \\ \times \quad 7 \\ \hline 266 \\ 5 \end{array}$ <p>The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.</p> <p>If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.</p>	<p>56×27 is approximately $60 \times 30 = 1800$.</p> <table border="1" data-bbox="1816 376 2107 576"> <tr><td>x</td><td>20</td><td>7</td><td></td></tr> <tr><td>50</td><td>1000</td><td>350</td><td>1350</td></tr> <tr><td>6</td><td>120</td><td>42</td><td>162</td></tr> <tr><td></td><td></td><td></td><td>1512</td></tr> <tr><td></td><td></td><td></td><td>1</td></tr> </table> <table border="1" data-bbox="1816 616 2107 815"> <tr><td>x</td><td>50</td><td>6</td><td></td></tr> <tr><td></td><td>20</td><td>7</td><td></td></tr> <tr><td></td><td>1000</td><td>350</td><td>1350</td></tr> <tr><td></td><td>120</td><td>42</td><td>162</td></tr> <tr><td></td><td></td><td></td><td>1512</td></tr> <tr><td></td><td></td><td></td><td>1</td></tr> </table> <p>56×27 is approximately $60 \times 30 = 1800$.</p> $\begin{array}{r} 56 \\ \times \quad 27 \\ \hline 1000 \quad 50 \times 20 = 1000 \\ 120 \quad 6 \times 20 = 120 \\ 350 \quad 50 \times 7 = 350 \\ 42 \quad 6 \times 7 = 42 \\ \hline 1512 \\ 1 \end{array}$ <p>56×27 is approximately $60 \times 30 = 1800$.</p> $\begin{array}{r} 56 \\ \times \quad 27 \\ \hline 1120 \quad 56 \times 20 \\ 392 \quad 56 \times 7 \\ \hline 1512 \\ 1 \end{array}$	x	20	7		50	1000	350	1350	6	120	42	162				1512				1	x	50	6			20	7			1000	350	1350		120	42	162				1512				1
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Multiplication

N	R	Y1	Y2	Y3	Y4 Stage 4 by the end of stage 4	Y5	Y6 Stage 5
Through practical activities in meaningful contexts		Doubles up to 10 + 10	double by partitioning and recombining →				
Grouping objects in twos or threes, then adding groups of the same number		Counting in 2s, 5's 10s	Doubles to 20+20 Multiples of 5 to 50 Doubles of multiples of 10 to 100 Counting in 2s, 10s + 5s. Reading $4 \times 2 = 8$ $\Delta \Delta \Delta \Delta$ $\Delta \Delta \Delta \Delta$ or $2 \times 4 = 8$ $\Delta \Delta$ $\Delta \Delta$ $\Delta \Delta$ $\Delta \Delta$	understand multiplication as repeated addition 15×3 15 added together 3 times $15 + 15 + 15$ 32×3 $30 \times 3 = 90$ $2 \times 3 = 6$ $90 + 6 = 96$	$\begin{pmatrix} 143 \times 2 \\ 100 \times 2 \\ 40 \times 2 \\ 3 \times 2 \end{pmatrix}$	$\begin{pmatrix} 487 \times 2 \\ 400 \times 2 \\ 80 \times 2 \\ 7 \times 2 \end{pmatrix}$	$\begin{pmatrix} 3620 \times 2 \\ 3000 \times 2 \\ 600 \times 2 \\ 20 \times 2 \end{pmatrix}$
counting in 2s, 10s					Stage 1 $200 + 80 + 6 = 286$	$800 + 160 + 14 = 974$	$6000 + 1200 + 40 = 7240$
Doubles up to 5 + 5					Stage 2 grid method →		
				<ul style="list-style-type: none"> Record + as x and vice versa. 	$\begin{array}{r} 23 \times 8 \\ \times \quad 20 \quad 3 \\ 8 \quad \color{red}{160} \quad \color{red}{24} \end{array}$	Stage 5 grid method 2 digit by 2 digit	$\begin{array}{r} 372 \times 24 \\ \times \quad 300 \quad 70 \quad 2 \\ 20 \quad \color{red}{6000} \quad \color{red}{1400} \quad \color{red}{40} \\ 4 \quad \color{red}{1200} \quad \color{red}{280} \quad \color{red}{8} \end{array}$
				<ul style="list-style-type: none"> Record using x and = signs Understand 3×2 as $2 + 2 + 2$ - 	<ul style="list-style-type: none"> Record + as x and vice versa. 	$\begin{array}{r} 72 \times 38 \\ \times \quad 70 \quad 2 \\ 30 \quad \color{red}{2100} \quad \color{red}{60} \\ 8 \quad \color{red}{560} \quad \color{red}{16} \end{array}$	$6000 + 1400 + 1200 + 280 + 40 + 8 = 8928$
				Multiply 1 or 2 digit nos by 10 or 100 Family facts for x / ÷	$160 + 24 = 184$ $2100 + 560 + 60 + 16 = 2736$		
					Decimals →		
					$\begin{array}{r} 346 \times 9 \\ \times \quad 300 \quad 40 \quad 6 \\ 9 \end{array}$	$\begin{array}{r} 4.9 \times 3 \\ \times \quad 4 \quad 0.9 \\ 3 \quad \color{red}{12} \quad \color{red}{2.7} \end{array}$	$\begin{array}{r} 4.92 \times 3 \\ \times \quad 4 \quad 0.9 \quad 0.02 \\ 3 \quad \color{red}{12} \quad \color{red}{2.7} \quad \color{red}{0.06} \end{array}$
					Multiply by 10 and 100 numbers to a 1000	$12 + 2.7 = 14.7$	$12 + 2.7 + 0.06 = 14.76$
					Stage 3 expanded Short multiplication vertically	$\begin{array}{r} 346 \\ \underline{9} \times \\ 54 \end{array} \quad (6 \times 9)$	$\begin{array}{r} 4346 \\ \underline{8} \times \\ 48 \end{array} \quad (6 \times 8)$

Multiplication

$$\begin{array}{r} 23 \text{ 20 + 3} \\ \underline{7} \times 7 \\ 21 \quad (3 \times 7) \\ \underline{140} \quad (20 \times 7) \\ 161 \end{array}$$

leading to:

$$\begin{array}{r} 23 \\ \underline{7} \times \\ \underline{161} \\ 2 \end{array}$$

$$\begin{array}{r} 360 \quad (40 \times 9) \\ \underline{2700} \quad (300 \times 9) \\ 3114 \end{array}$$

leading to:

$$\begin{array}{r} 72 \\ \underline{38} \times \\ 576 \quad (72 \times 8) \\ \underline{2160} \quad (72 \times 30) \\ 2736 \end{array}$$

$$\begin{array}{r} 320 \quad (40 \times 8) \\ 2400 \quad (300 \times 8) \\ \underline{32000} \quad (4000 \times 8) \\ 34768 \end{array}$$

leading to:

$$\begin{array}{r} 352 \\ \underline{27} \times \\ 2464 \\ \underline{7040} \\ 9504 \end{array}$$

Units first
(relies on mental calculation strategies)

Division

(the strategy recommend partitioning (of multiples eg $72 \div 3 = 60 + 12 \div 3$)- stage 1 and 2 for short division. This policy uses repeated subtraction moving on to chunking)

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in $18 \div 3 = 6$, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10×10 , recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Note: It is important that children’s mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

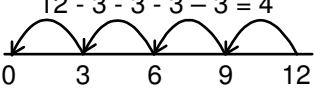
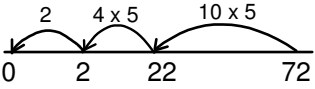
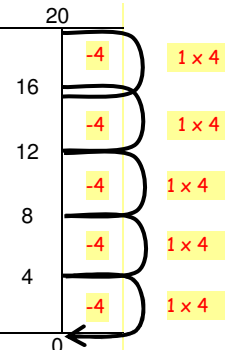
- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

Examples:

Single-digit division	1. No remainder, no carrying	$3 \overline{)69}$	$3 \overline{)264}$
	2. Remainder, no carrying	$3 \overline{)68}$	
	3. No remainder, carrying	$3 \overline{)45}$	
	4. Remainder, carrying	$3 \overline{)47}$	
	5. Placing of the quotient	$7 \overline{)287}$	
	6. Noughts in quotient	$4 \overline{)816}$	$8 \overline{)5608}$
	7. No remainder	$32 \overline{)64}$	$31 \overline{)93}$
Two digit division	8. Similar but remainder	$13 \overline{)29}$	$31 \overline{)97}$
	9. Quotient not so apparent	$22 \overline{)56}$	$41 \overline{)92}$
	10. Placing the quotient	$21 \overline{)126}$	$32 \overline{)224}$
	11. No remainder	$21 \overline{)483}$	$32 \overline{)224}$
	12. Remainder	$33 \overline{)718}$	
	13. Noughts in quotient	$17 \overline{)6834}$	
	14. Divisors like 29, 39, 48		
	15. Divisors like 45, 37, 24, 56		

Division

(the strategy recommend partitioning (of multiples eg $72 \div 3 = 60 + 12 \div 3$)- stage 1 and 2 for short division. This policy uses repeated subtraction moving on to chunking)

Stage 1 Grouping and sharing	Stage 2 Repeated subtraction	Stage 3 Expanded short division leading to compact short division (by end of year 4)	Stage 4 Long Division (by end of year 5)
<p>Division as Grouping A bag of 6 sweets, how many children can have 2 sweets each. The division sign (\div) 'divided into groups of'</p> <p>Division as Sharing Share equally Share a bag of 6 sweets between 2 children – one for you, one for me...</p>	<p>Counting in multiples Use hands: How many groups of 5 in 15? How many 5s have been counted? How many more 5s do we need to reach 25? Use number square to demonstrate counting in multiples. Using a number line: $12 \div 3 = 4$ $12 - 3 - 3 - 3 - 3 = 4$  $72 \div 5$ Can we subtract 10 lots of 5? How many other lots of 5 can we subtract?  Turn number line around </p>	<p>$97 \div 9$</p> $\begin{array}{r} 9 \overline{)97} \\ - 90 \\ \hline 7 \end{array} \quad 9 \times 10$ <p>Answer: 10 R7</p> <hr/> <p>$6 \overline{)196}$</p> $\begin{array}{r} - 60 \\ 136 \\ - 60 \\ 76 \\ - 60 \\ 16 \\ - 12 \\ 4 \end{array} \quad \begin{array}{l} 6 \times 10 \\ 6 \times 10 \\ 6 \times 10 \\ 6 \times 2 \end{array}$ <p>Answer: 32 R4</p> <p>To find $196 \div 6$, we start by multiplying 6 by 10, 20, 30, ... to find that $6 \times 30 = 180$ and $6 \times 40 = 240$. The multiples of 180 and 240 trap the number 196. This tells us that the answer to $196 \div 6$ is between 30 and 40.</p> <p>Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4.</p> $\begin{array}{r} 6 \overline{)196} \\ - 180 \\ 16 \\ - 12 \\ 4 \end{array} \quad \begin{array}{l} 6 \times 30 \\ 6 \times 2 \end{array}$ <p>Answer: 32 R4</p> <p>The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.</p>	<p>How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As $24 \times 20 = 480$ and $24 \times 30 = 720$, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.</p> $\begin{array}{r} 24 \overline{)560} \\ - 480 \\ \hline 80 \\ 3 \overline{)80} \\ - 72 \\ \hline 8 \end{array} \quad \begin{array}{l} 24 \times 20 \\ 24 \times 3 \end{array}$ <p>Answer: 23 R 8</p> <p>In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.</p> $\begin{array}{r} 23 \\ 24 \overline{)560} \\ - 480 \\ \hline 80 \\ - 72 \\ \hline 8 \end{array}$ <p>Answer: 23 R 8</p>

Division

(the strategy recommend partitioning (of multiples eg $72 \div 3 = 60 + 12 \div 3$)- stage 1 and 2 for short division. This policy uses repeated subtraction moving on to chunking)

N	R	Y1	Y2	Y3	Y4	Y5	Y6
		Halving up to 10	Halving by partitioning and recombining				
		understanding $8 \div 2$ as half of 8	Understand division as grouping (repeated subtraction) Interpret $8 \div 2$ as how many 2s make 8? $\Delta \Delta / \Delta \Delta / \Delta \Delta / \Delta \Delta$ and '8 put into groups of 2'		$\begin{array}{r} 72 \div 2 \\ 70 \div 2 = 35 \\ 2 \div 2 = 1 \end{array}$	$\begin{array}{r} 358 \div 2 \\ 300 \div 2 = 150 \\ 50 \div 2 = 25 \\ 8 \div 2 = 4 \end{array}$	$\begin{array}{r} 3476 \div 2 \\ 3000 \div 2 \\ 400 \div 2 \\ 70 \div 2 \\ 6 \div 2 \end{array}$
		Through practical activities in meaningful contexts Grouping objects equally 10 grouped into 2s *How many groups?*	Halving of numbers up to 20 Halves of multiples of 10 to 100	Use multiplication facts and count up/back $20 \div 4 = 5$ count up/back in 4s	$35 + 1 = 36$ Concrete 'acting out' of repeated subtraction	$150 + 25 + 4 = 179$	$1500 + 200 + 35 + 3 = 1738$
		Grouping 2,5,10 Represent number stories using \div sign.	Record using \div and = signs Know related division facts for 2 x 5 x 10 x tables Recognise relationship between x and \div Include calculations with remainders	<p>20 16 12 8 4 0</p> <p>remainders i.e. $21 \div 4 = 5 \text{ r } 1$</p> <p>21 17 13 9 5 1 0</p> <p>Inverse of x</p>	repeated subtraction (chunking)		
					$\begin{array}{r} 72 \div 5 \\ 14 \text{r} 2 \\ 5 \overline{)72} \\ \underline{-20} \quad (4 \times 5) \\ 52 \\ \underline{-50} \quad (4 \times 5) \\ 2 \\ \underline{-20} \quad (4 \times 5) \\ 32 \\ \underline{-30} \quad (4 \times 5) \\ 2 \end{array}$	$\begin{array}{r} 256 \div 7 \\ 36 \text{r} 4 \\ 7 \overline{)256} \\ \underline{-210} \quad (30 \times 7) \\ 46 \\ \underline{-42} \quad (6 \times 7) \\ 4 \end{array}$	$\begin{array}{r} 977 \div 36 \\ 27 \text{r} 5 \\ 36 \overline{)977} \\ \underline{-720} \quad (20 \times 36) \\ 257 \\ \underline{-180} \quad (5 \times 36) \\ 77 \\ \underline{-72} \quad (2 \times 36) \\ 5 \end{array}$
					bracketed numbers represent how many groups	Decimals	
						Express remainder as fraction.	$7 \overline{)87.5}$
					Remainders rounded depending on context		

Vocabulary

	Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition and subtraction	<p>add, more, and make, sum, total altogether score double one more, two more, ten more... how many more to make... ? how many more is... than... ? take (away), leave how many are left/left over? how many have gone? one less, two less... ten less... how many fewer is... than... ? difference between is the same as</p>	<p>+, □□plus, near double, how much more is...? -, subtract, minus how much less is...? half, halve =, □□equals, sign Tens, ones</p> <p>Count up Number sentence digit</p>	<p>addition, one hundred more, subtraction, one hundred less, tens boundary ones, units, tens</p>	<p>hundreds boundary</p> <p>partition 1 digit 2 digit 3 digit inverse</p>	<p>Increase, decrease, inverse</p> <p>Thousands 4 digits decimal point decimal place tenths/hundredths negative</p>	<p>units boundary, tenths boundary</p> <p>ten thousands million</p>	<p>integer</p>
Multiplication and division		<p>double, halve share, left, left over</p> <p>half</p> <p>near double</p> <p>count in 2s,5s,10s</p> <p>groups of</p>	<p>lots of, groups of x, times, multiply, multiplied by multiple of once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column share equally one each, two each, three each... group in pairs, threes... tens equal groups of ÷, divide, divided by, divided into</p> <p>multiplication</p>	<p>multiplication, product</p> <p>left left over remainder round up/down grid row column</p>	<p>factor, quotient, divisible by inverse</p> <p>divisor</p>		<p>dividend</p>

